ELECTRICAL PHENOMENA ON EXPOSING A CABLE TO NEUTRONS AND GAMMA RAYS

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ABSTRACT: The discussion concerns the effects of ionizing radiation on pulse formation between conductors in a cable.

1. Neutrons and γ-rays produce electric fields in insulators mainly because of polarization [1–5]. The potential difference between sheath and core in a coaxial cable is considered first statically (all quantities independent of time).

The insulator (dielectric constant ε) fills the space between two concentric conducting cylinders with $r_1 < r_2$ (Fig. 1). The inner conductor is shown with oblique hatching ($0 < r < r_1$), while the outer metal sheath ($r = r_2$) is thin enough for its absorption to be neglected. The neutrons or γ-rays fall normally on the cable from the left.

Let δ be the mean range of a Compton electron in the insulator in the line of motion of a γ-ray. A positive ion remains where the electron was ejected, and the resulting electric dipole has a moment whose components are $p_x = -e_0\delta$, $p_y = 0$, in which e_0 is the absolute electronic charge.

A neutron produces a recoil proton, which forms with the negatively charged ion a dipole of opposite sign, $p_x = e_0\Delta$, $p_y = 0$, in which Δ is the range of the recoil proton. In what follows we consider only the effects of γ-rays, as the neutron effect can be deduced by changing the sign and replacing δ by Δ.

Let J be the number of γ-rays passing through 1 cm² to the left of the cable perpendicular to the x-axis. At a point whose coordinates are x and y we have

$$J(x, y) = J \exp \frac{-L(x, y)}{\lambda_0}, \tag{1.1}$$

in which λ_0 is the range of the γ-rays in the insulator and L(x, y) is the distance from the point of γ-ray entry to (x, y). Figure 1 gives

$$L(x, y) = \sqrt{r_2^2 - y^2} + x. \tag{1.2}$$

The volume density n_γ of the γ-rays absorbed in the material is

$$n_\gamma = -\frac{\partial J(x, y)}{\partial x} = \frac{J}{\lambda_0} \exp \frac{-L(x, y)}{\lambda_0} \frac{\partial L}{\partial x} = \frac{J}{\lambda_0} \exp \frac{-L(x, y)}{\lambda_0}, \tag{1.3}$$

since (1.2) gives $\partial L/\partial x = 1$. Usually, $\lambda_0 \gg 2r_2$, so we can replace (1.3) with adequate accuracy by

$$n_\gamma = \frac{J}{\lambda_0}\left[1 - \frac{\sqrt{r_2^2 - y^2} + x}{\lambda_0}\right]. \tag{1.4}$$

In the shadow region (horizontal hatching in Fig. 1) the γ-ray flux is

$$J(x, y) = J \exp\left[-\frac{L - L_1}{\lambda_0} - \frac{L_1}{\lambda_1}\right] = J \exp\left[-\frac{L}{\lambda_0} + L_1\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_1}\right)\right]$$

$$L_1 = 2\sqrt{r_1^2 - y^2} \tag{1.5}$$

in which λ_1 is the γ-ray range in the core material and L_1 is the γ-ray path length in the core. Also, $\partial L_1/\partial x = 0$, so the absorbed γ-ray density in the shadow region is

$$n_\gamma = -\frac{\partial J(x, y)}{\partial x} = \frac{J}{\lambda_0} \exp\left[-\frac{L}{\lambda_0} + L_1\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_1}\right)\right]. \tag{1.6}$$

We assume that $\lambda_1 \gg 2r_1$; then for the shadow region

$$n_\gamma = \frac{J}{\lambda_0}\left[1 - \frac{\sqrt{r_2^2 - y^2} + x}{\lambda_0} + 2\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_1}\right)\sqrt{r_1^2 - y^2}\right]. \tag{1.7}$$

Let there be one Compton electron per N absorbed γ-rays. The volume density of the Compton electrons is

$$n_e = n_\gamma / N. \tag{1.8}$$

The Compton electron travels a distance δ along the x-axis from the point of ejection; we assume that $\delta \ll r_2$, and then the entire dielectric is polarized. The vector P_0 for the initial polarization has the components

$$P_{0x} = -e_0\delta n_e, \qquad P_{0y} = 0. \tag{1.9}$$

This spatially inhomogeneous polarization gives a charge density different from zero:

$$\rho_0 = -\operatorname{div} \mathbf{P}_0. \tag{1.10}$$

From (1.4), (1.7), and (1.9) we find that the bulk charge density is constant throughout the dielectric and is

$$\rho_0 = \text{const} = -C, \qquad C = \frac{e_0\delta}{N\lambda_0^2} J \tag{1.11}$$

The field equations are as follows for a homogeneous dielectric:

$$\operatorname{div} \mathbf{D} = 4\pi\rho_0, \quad \mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{E} = -\nabla\psi, \quad \Delta\psi = -\frac{4\pi}{\varepsilon}\rho_0. \tag{1.12}$$

and these must be solved subject to the conditions that the circles $r = r_1$ and $r = r_2$ are equipotentials. For a solution of the form ψ(r) we have

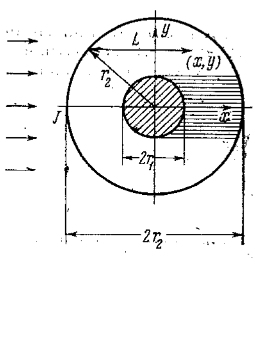
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi}{dr}\right) = \frac{4\pi}{\varepsilon} C. \tag{1.13}$$

Then

$$\frac{d\psi}{dr} = -E_r = \frac{2\pi}{\varepsilon} Cr - \frac{C_1}{r},$$

$$\psi = \frac{\pi}{\varepsilon} Cr^2 + C_1 \ln r + C_2, \tag{1.14}$$

in which C_1 and C_2 are arbitrary constants. To find C_1 we must calcu-



late the total charge at $r = r_1$. The surface of discontinuity in the polarization vector has a surface charge of density

$$\sigma_0 = -(P_{2n} - P_{1n}), \tag{1.15}$$

and the vector along the normal is directed from medium 1 into medium 2. In this case medium 2 is the insulator, while medium 1 is the metal, which has no polarization. The direction of the normal coincides with the radius vector r, so at the circle $r = r_1$

$$\sigma_0 = -P_{0r}. \tag{1.16}$$

As $r = r_1$ we have $x = r_1 \cos\varphi$, $y = r_1 \sin\varphi$. The shadow region corresponds to the range of angles $-\pi/2 < \varphi < +\pi/2$, while the interval $\pi/2 < \varphi\ 3\pi/2$ is outside the shadow. As $P_{0r} = P_{0x} \cos\varphi$, we get from (1.4), (1.7), and (1.9) that for $-\pi/2 < \varphi < +\pi/2$

$$\sigma_0(\varphi) = \lambda_0 C\left[1 - \frac{\sqrt{r_2^2 - (r_1 \sin\varphi)^2} + r_1\cos\varphi}{\lambda_0} + 2\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_1}\right) r_1 \cos\varphi\right]\cos\varphi \tag{1.17}$$

and for $^1/_2\pi < \varphi < {^3/_2}\,\pi$

$$\sigma_0(\varphi) = \lambda_0 C\left[1 - \frac{\sqrt{r_2^2 - (r_1 \sin\varphi)^2} + r_1\cos\varphi}{\lambda_0}\right]\cos\varphi. \tag{1.18}$$

We see that (1.17) gives $\sigma_0 > 0$, while (1.18) gives $\sigma_0 < 0$, as should be the case from physical considerations. Let q_0 be the total charge (per cm of cable length) adjoining $r = r_1$:

$$q_0 = r_1 \int_{-{^1/_2}\pi}^{{^3/_2}\pi} \sigma_0(\varphi)\, d\varphi = r_1\left[\int_{-{^1/_2}\pi}^{{^1/_2}\pi} \sigma_0(\varphi)\, d\varphi + \int_{{^1/_2}\pi}^{{^3/_2}\pi} \sigma_0(\varphi)\, d\varphi\right]. \tag{1.19}$$

We make the substitution $\varphi \to \pi + \varphi$ in the second integral to get

$$q_0 = -\frac{\pi r_1^2 \lambda_0}{\lambda_1} C. \tag{1.20}$$

From (1.12) and (1.14) we get the radial component of the induction vector as

$$D_r = -\varepsilon\left[\frac{2\pi}{\varepsilon} Cr + \frac{C_1}{r}\right]. \tag{1.21}$$

Gauss's theorem gives

$$\oint D_n\, ds = 4\pi q, \tag{1.22}$$

in which the integral is taken over a closed surface and q is the charge within that surface. In the present case, the integral is to be taken over the circle $r = r_1$, and q is replaced by (1.20):

$$-2\pi r_1 \varepsilon\left[\frac{2\pi}{\varepsilon} C r_1 + \frac{C_1}{r_1}\right] = 4\pi\left(-\frac{\pi r_1^2 \lambda_0}{\lambda_1} C\right). \tag{1.23}$$

Then

$$C_1 = \frac{2\pi}{\varepsilon} r_1^2 \lambda_0 C\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right). \tag{1.24}$$

This C_1 is substituted into (1.14) to get for $V = \psi(r_2) - \psi(r_1)$, the potential difference between core and sheath, that

$$V = \frac{B}{\varepsilon} J,$$

$$B = \pi r_1^2 a\left[\left(\frac{r_2}{r_1}\right)^2 - 1 + 2\left(\frac{\lambda_0}{\lambda_1} - 1\right)\ln\left(\frac{r_2}{r_1}\right)\right]$$

$$a = \frac{e_0 \delta}{N \lambda_0^2} \tag{1.25}$$

2. Consider now the effects of a conductivity σ in the insulator and of time variation in the γ-ray flux. This σ has a complicated relation to the radiation intensity. We have assumed that there is only slight γ-ray absorption in the cable, so $\sigma = \sigma(t)$ and is the same at all points in the insulator. Similarly, we assume that the dielectric constant $\varepsilon = \varepsilon(t)$ is a known function of time.

Let R be the load resistance joining the outer and inner conductors at one end of the cable. The load current I(t) flows from the sheath to the core and is

$$I(t) = \frac{V(t)}{R} \tag{2.1}$$

in which V(t) is the potential difference between those conductors. We have seen in section 1 that the γ-ray flux generates a charge density ρ_0 uniform throughout the insulator, so the resulting conduction current j will be radial and will be independent of φ. Let $\rho_1(r, t)$ be the bulk charge density due to the conduction. Then ρ_1 and $j = \sigma E$ are related by

$$\frac{\partial \rho_1}{\partial t} + \sigma \operatorname{div} \mathbf{E} = 0. \tag{2.2}$$

The field equation

$$\operatorname{div} D = \varepsilon \operatorname{div}\mathbf{E} = 4\pi\rho, \tag{2.3}$$

contains the total volume charge density ρ, whose derivative with respect to time can be put as the sum of two terms:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_0}{\partial t}. \tag{2.4}$$

The result is

$$\frac{\partial \rho}{\partial t} + \frac{\rho}{\tau(t)} = \frac{\partial \rho_0}{\partial t}, \qquad \tau(t) = \frac{\varepsilon(t)}{4\pi\sigma(t)}. \tag{2.5}$$

We extend (1.11) to a time-varying γ-ray flux and put

$$\frac{\partial \rho_0}{\partial t} = -aJ(t), \qquad a = \frac{e_0 \delta}{N \lambda_0^2}. \tag{2.6}$$

A difference from (1.11) is that here and subsequently in sections 2 and 3 we denote by J(t) the γ-ray flux through 1 cm² in 1 sec. We assume that irradiation of the cable starts at t = 0. Then obviously $\rho(t = 0) = 0$, and (2.5) and (2.6) give

$$\rho(t) = -a \exp\left[-\int_0^t \frac{d\alpha}{\tau(\alpha)}\right] \int_0^t J(t') \exp\left[\int_0^{t'} \frac{d\beta}{\tau(\beta)}\right] dt'. \tag{2.7}$$

The charge density is again constant over the volume of the dielectric. Poisson's equation is as follows for $\psi(r_1 t)$:

$$\Delta\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = -\frac{4\pi}{\varepsilon(t)}\rho(t), \tag{2.8}$$

and the solution is

$$\frac{\partial \psi}{\partial r} = -E_r = \frac{r}{2} f(t) + \frac{A(t)}{r},$$

$$\psi(r, t) = \frac{r^2}{4} f(t) + A(t)\ln r + A_1(t), \tag{2.9}$$

$$f(t) = -\frac{4\pi a}{\varepsilon(t)} \exp\left[-\int_0^t \frac{d\alpha}{\tau(\alpha)}\right] \int_0^t J(t') \exp\left[\int_0^{t'} \frac{d\beta}{\tau(\beta)}\right] dt'. \tag{2.10}$$

Here A(t) and $A_1(t)$ are certain functions of time; $A_1(t)$ does not affect the physical results, so we can put it as zero. Then (2.9) gives

$$V(t) = \psi(r_2, t) - \psi(r_1, t) = \frac{1}{4} f(t)(r_2^2 - r_1^2) + A(t)\ln\left(\frac{r_2}{r_1}\right). \tag{2.11}$$

The total charge q(t) per unit length of internal cylinder ($r = r_1$) is found by setting $r = r_1$ in (2.9) and using (1.22):

$$q(t) = -\frac{\varepsilon(t)}{2}\left[\frac{r_1^2}{2} f(t) + A(t)\right]. \tag{2.12}$$

The total current $I_1(t)$ per unit length of internal cylinder is $2\pi\sigma(t) \times$ $\times E_r(r = r_1)$ or, from (2.9),

$$I_1(t) = -\sigma(t)\, 2\pi\left[\frac{r_1^2}{2} f(t) + A(t)\right] = \frac{q(t)}{\tau(t)}. \tag{2.13}$$

Let L be the length of the cable. We assume that the charge carried by I(t) through the load R is distributed uniformly along the entire length of the cable, and so

$$\frac{dq}{dt} = \frac{I(t)}{L} - I_1(t) + \frac{dq_0}{dt}. \tag{2.14}$$

The term dq_0/dt takes account of the change in surface charge adjoining the cylinder $r = r_1$. The extension of (1.20) to the transient case is

$$\frac{dq_0}{dt} = -bJ(t), \qquad b = \pi r_1^2 \frac{\lambda_0}{\lambda_1} a. \tag{2.15}$$

Here J(t) and a are as in (2.6). We use (2.1) and (2.11)-(2.13) to get for V(t) that

$$\frac{dV}{dt} + \frac{V}{T(t)} = \frac{B}{\varepsilon(t)} J(t), \tag{2.16}$$

where B is defined by (1.25) and

$$\frac{1}{T(t)} = \frac{1}{\tau(t)} + \frac{2\ln(r_2/r_1)}{\varepsilon(t) RL} + \frac{d\ln\varepsilon}{dt}. \tag{2.17}$$

If $V(t = 0) = 0$, the solution to (2.16) is

$$V(t) = B\exp\left[-\int_0^t \frac{d\alpha}{T(\alpha)}\right]\int_0^t \frac{J(t')}{\varepsilon(t')}\exp\left[\int_0^{t'}\frac{d\beta}{T(\beta)}\right]dt'. \tag{2.18}$$

There is no difficulty in taking account of an external emf $\mathscr{E}(t)$ connected as shown in Fig. 2. Let R_1 be the internal resistance of the source; then (2.1) is replaced by

$$I(t) = \frac{V(t) - \mathscr{E}(t)}{R + R_1}. \tag{2.19}$$

The above arguments then give for V(t) that

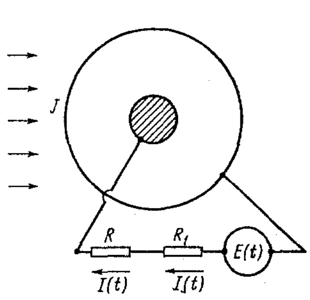
$$\frac{dV}{dt} + \frac{V}{T_1(t)} = \frac{B}{\varepsilon(t)} J(t) + \frac{2\ln(r_2/r_1)}{\varepsilon(t)(R + R_1)L}\mathscr{E}(t), \tag{2.20}$$

in which

$$\frac{1}{T_1(t)} = \frac{1}{\tau(t)} + \frac{2\ln(r_2/r_1)}{\varepsilon(t)(R + R_1)L} + \frac{d\ln\varepsilon}{dt}. \tag{2.21}$$

We naturally assume that $\sigma(t = 0) = 0$, so at $t = 0$, we have $V(t = 0) =$ $= \mathscr{E}(t = 0) = \mathscr{E}_0$. Then

$$V(t) = \exp\left[-\int_0^t \frac{d\alpha}{T_1(\alpha)}\right]\Bigg\{\mathscr{E}_0 + \int_0^t \frac{1}{\varepsilon(t')}\left[BJ(t') + \frac{2\ln(r_2/r_1)}{L(R + R_1)}\mathscr{E}(t')\right]\exp\left[\int_0^{t'}\frac{d\beta}{T_1(\beta)}\right]dt'\Bigg\} \tag{2.22}$$



Usually $\mathscr{E}$ = const = $\mathscr{E}_0$, and then

$$V(t) = B\exp\left[-\int_0^t \frac{d\alpha}{T_1(\alpha)}\right]\int_0^t \frac{J(t')}{\varepsilon(t')}\exp\left[\int_0^{t'}\frac{d\beta}{T_1(\beta)}\right]dt' + \mathscr{E}_0\exp\left[-\int_0^t\frac{d\alpha}{T_1(\alpha)}\right]\left\{1 + \frac{2\ln(r_2/r_1)}{L(R + R_1)}\int_0^t \frac{1}{\varepsilon(t')}\exp\left[\int_0^{t'}\frac{d\beta}{T_1(\beta)}\right]dt'\right\}. \tag{2.23}$$

3. Consider a long cable in the quasi-stationary approximation. Let $\mathscr{L}$ and $\mathscr{R}$ be the self-inductance and resistance of unit length of cable. The x-axis lies along the axis of the cable, v(x, t) is the potential difference between the core and sheath, and c is the speed of light in vacuum. The voltage drop in an element dx is $(\partial v/\partial x)dx$ and has two components: the ohmic one $\mathscr{R}i(x,t)\,dx$ and the one due to self-induction $(\mathscr{L}/c^2)(\partial i/\partial t)\,dx$. Then

$$\frac{\partial v}{\partial x} = -\mathscr{R}i - \frac{\mathscr{L}}{c^2}\frac{\partial i}{\partial t}. \tag{3.1}$$

Let q(x, t) be the charge per unit length of internal cylinder. The conservation of charge is expressed by

$$\frac{\partial q}{\partial t} = -\frac{\partial i}{\partial x} - I_1(x, t) + \frac{\partial q_0}{\partial t}. \tag{3.2}$$

Here $I_1(x, t)$ is the current due to the conductivity $\sigma(x, t)$ per unit length of core. We extend (2.12) and (2.13) to write for a long cable that

$$q(x, t) = -\frac{\varepsilon(x, t)}{2}\left[\frac{r_1^2}{2} f(x, t) + A(x, t)\right],$$

$$I_1(x, t) = \frac{q(x, t)}{\tau(x, t)}, \quad \tau(x, t) = \frac{\varepsilon(x, t)}{4\pi\sigma(x, t)}, \tag{3.3}$$

where $f(x, t)$ is defined by a function analogous to (2.10):

$$f(x, t) = \frac{4\pi a}{\varepsilon(x, t)}\exp\left[-\int_0^t \frac{d\alpha}{\tau(x, \alpha)}\right] \times \int_0^t J(x, t')\exp\left[\int_0^{t'}\frac{d\beta}{\tau(x, \beta)}\right]dt' \tag{3.4}$$

We replace V(t) in (2.11) by $-v(x, t)$ to get

$$-v(x, t) = \frac{r_2^2 - r_1^2}{4} f(x, t) + A(x, t)\ln(r_2/r_1). \tag{3.5}$$

An obvious extension of (2.15) is

$$\frac{\partial q_0(x, t)}{\partial t} = -bJ(x, t). \tag{3.6}$$

Then (3.3)–(3.6) allow us to put (3.2) as

$$\frac{\partial v}{\partial t}+\left[\frac{1}{\tau(x,t)}+\frac{\partial \ln \varepsilon(x,t)}{\partial t}\right]v+$$

$$+\frac{2\ln(r_2/r_1)}{\varepsilon(x,t)}\frac{\partial i}{\partial x}=-B\frac{J(x,t)}{\varepsilon(x,t)} \quad (3.7)$$

System (3.1) and (3.7) is the basis for future calculations. Let c_1 be the local speed of a signal along the cable, C the capacity per unit length, and R^* the wave impedance:

$$c_1=\frac{c}{\sqrt{\mathscr{L}C}},\quad C=\frac{\varepsilon}{2\ln(r_2/r_1)},$$

$$R^*=\frac{1}{c}\sqrt{\frac{\mathscr{L}}{C}}=\frac{1}{c_1C}. \quad (3.8)$$

Consider the case in which $\varepsilon(x,t)$ = const = ε, $\sigma(x,t)$ = const = σ, $J(x,t) = J(t)$; then (3.1) and (3.7) give

$$\frac{\partial v}{\partial x}+\frac{\mathscr{L}}{c^2}\frac{\partial i}{\partial t}+\mathscr{R}i=0,$$

$$\frac{\partial v}{\partial t}+\frac{v}{\tau}+\frac{1}{C}\frac{\partial i}{\partial x}=-\frac{B}{\varepsilon}J(t)=F(t). \quad (3.9)$$

The left end of the cable is connected to a resister R_0 and the right end to a resistor R_1. Let L_0 be the length of the cable. The boundary conditions are

$$R_0 i(0,\ t)=-v(0,\ t),\quad x=0;$$

$$R_1 i(L_0,\ t)=v(L_0\ t),\quad x=L_0 \quad (3.10)$$

We take the initial conditions as zero:

$$i(x,\ 0)=0,\quad v(x,\ 0)=0. \quad (3.11)$$

The problem is solved via a Laplace transformation with respect to time, i.e., we pass from the original $\varphi(x,t)$ to

$$\varphi(x,t)\div\!\!\rightarrow \varphi_1(x,p)=\int_0^\infty \varphi(x,t)\,e^{-pt}\,dt. \quad (3.12)$$

The corresponding system for $i_1(x,p)$ and $v_1(x,p)$ is

$$\frac{dv_1}{dx}+\left(\mathscr{R}+\frac{\mathscr{L}p}{c^2}\right)i_1=0,$$

$$\left(p+\frac{1}{\tau}\right)v_1+\frac{1}{C}\frac{di_1}{dx}=F_1(p). \quad (3.13)$$

The boundary conditions for these are

$$R_0 i_1(0,\ p)=-v_1(0,\ p),\ R_1 i_1(L_0,\ p)=v_1(L_0,\ p). \quad (3.14)$$

The general solution to (3.13) is

$$i_1(x,p)=A_1\operatorname{ch}\alpha x+A_2\operatorname{sh}\alpha x,$$

$$v_1(x,p)=-R^*\left(\frac{p+1/\tau_1}{p+1/\tau}\right)^{1/2}\times$$

$$\times(A_1\operatorname{sh}\alpha x+A_2\operatorname{ch}\alpha x)+\frac{F_1(p)}{p+1/\tau}. \quad (3.15)$$

Here A_1 and A_2 are arbitrary constants, while

$$\tau_1=\frac{\mathscr{L}}{c^2\mathscr{R}},\quad \alpha=\frac{1}{c_1}\left[\left(p+\frac{1}{\tau}\right)\left(p+\frac{1}{\tau_1}\right)\right]^{1/2} \quad (3.16)$$

We take the part of the root that gives $\alpha > 0$ for $p > 0$. Substitution of (3.15) into (3.14) gives

$$v_1(L_0,p)=R_1\frac{F_1(p)}{p+1/\tau}\times$$

$$\times\frac{\operatorname{ch}\alpha L_0-1+(R_0/R^*)P\operatorname{sh}\alpha L_0}{(R_0+R_1)\operatorname{ch}\alpha L_0+[(R_0R_1/R^*)P+R^*P^{-1}]\operatorname{sh}\alpha L_0}$$

$$P=[(p+1/\tau)/(p+1/\tau_1)]^{1/2} \quad (3.17)$$

We put

$$y=\frac{t}{\tau},\quad q=p\tau,\quad \beta=\frac{L_0}{c_1\tau},$$

$$\mu=\alpha L_0,\quad K=\left(\frac{q+1}{q+\tau/\tau_1}\right)^{1/2}. \quad (3.18)$$

Reverting to the originals, we have

$$v(L_0,t)=\frac{R_1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{qy}\frac{F_1(p)}{q+1}\times$$

$$\times\frac{[\operatorname{ch}\mu-1+(R_0/R^*)K\operatorname{sh}\mu]\,dq}{(R_0+R_1)\operatorname{ch}\mu+[(R_0R_1/R^*)K+R^*K^{-1}]\operatorname{sh}\mu}. \quad (3.19)$$

As sh x is odd, the integrand is a function of one sheet. The integral of (3.19) is calculated along a vertical line in a plane of q to the right of all poles in the integrand.

As $\beta \ll 1$ for a sufficiently short cable, the most substantial contribution to the integral comes from poles with a finite value of q. We expand the numerator and denominator in (3.19) in powers of β to get

$$v(L_0,t)=\frac{1}{1+\gamma}\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{pt}\frac{F_1(p)\,dp}{p+T_1^{-1}}=$$

$$=-\frac{B}{(1+\gamma)\varepsilon}e^{-t/T_1}\int_0^t J(t')\,e^{t'/T_1}dt'. \quad (3.20)$$

Here

$$\frac{1}{T_1}=\frac{1}{1+\gamma}\left[\frac{2\ln(r_2/r_1)}{\varepsilon L_0 R}+\frac{1}{\tau}\left(1+\gamma\frac{\tau}{\tau_1}\right)\right],$$

$$\gamma=\frac{(R^*)^2}{R_0R_1},\quad \frac{1}{R}=\frac{1}{R_0}+\frac{1}{R_1}. \quad (3.21)$$

We put ε = const, τ = const in (2.18) to get

$$V(t)=\frac{B}{\varepsilon}e^{-t/T}\int_0^t J(t')\,e^{t'/T}dt',$$

$$\frac{1}{T}=\frac{1}{\tau}+\frac{2\ln(r_2/r_1)}{\varepsilon LR}. \quad (3.22)$$

This solution coincides with (3.20) if $\gamma \ll 1$. The example shows that, for (2.18) to apply, we must have not only $L_0 \ll c_1\tau$ but also $R^* \ll \sqrt{R_0R_1}$. In particular, if one end is open-circuited (e.g., $R_0 = \infty$), then for $L_0 \ll c_1\tau$ we can use (2.18), since wave effects are then unimportant.

Without loss of generality we can consider only pulse irradiation:

$$J(t)=\delta(t),\qquad F_1(p)=-\frac{B}{\varepsilon}. \quad (3.23)$$

Here $\delta(t)$ is a delta function. We envisage a long cable: $\beta = L_0/c_1\tau \gg 1$. In (3.19) we replace ch μ and sh μ and $e^{\mu}/2 \gg 1$ to get

$$v(L_0\to\infty,\ t)=-\frac{B}{\varepsilon}\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{pt}}{p+1/\tau}\frac{dp}{1+(R^*/R_1)P^{-1}}. \quad (3.24)$$

This formula does not contain R_0, i.e., $v(L_0,t)$ is independent of the load at the left end for $\beta \gg 1$ and $i(x,t)$ becomes zero far from the right end. For simplicity we assume that $R_1 = R^*$ (matched load). For $\tau_1 > \tau$ we put

$$p+\frac{1}{\tau_1}=s,\qquad \alpha=\frac{1}{2}\left(\frac{1}{\tau}-\frac{1}{\tau_1}\right)>0.$$

We put the integral of (3.24) as

$$v(L_0\to\infty,t)=-\frac{B}{2\alpha\varepsilon}e^{-t/\tau_1}\left\{\frac{1}{2\pi i}\int\frac{\alpha e^{st}\,ds}{\sqrt{s^2+2\alpha s}}-\right.$$

$$\left.-\frac{1}{2\pi i}\int e^{st}\frac{s+\alpha-\sqrt{s^2+2\alpha s}}{\sqrt{s^2+2\alpha s}}ds\right\}=$$

$$=-\frac{B}{2\varepsilon}e^{-t/\tau_1}\{e^{-\xi}I_0(\xi)-e^{-\xi}I_1(\xi)\}=$$

$$=-\frac{B}{2\varepsilon}\exp\left[-\frac{1}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)t\right]\{I_0(\xi)-I_1(\xi)\},$$

$$\xi=\frac{1}{2}\left(\frac{1}{\tau}-\frac{1}{\tau_1}\right)t>0, \qquad (3.25)$$

in which $I_0(\xi)$ and $I_1(\xi)$ are Bessel functions of imaginary argument.

If $\tau_1<\tau$ we put

$$p+1/\tau=s,\quad \alpha={}^1\!/_2\,(1/\tau_1-1/\tau)>0,$$

The integral of (3.24) is transformed to

$$v(L_0\to\infty,t)=-\frac{B}{2\alpha\varepsilon}e^{-t/\tau}\left\{\frac{1}{2\pi i}\int\frac{\alpha e^{st}\,ds}{\sqrt{s^2+2\alpha s}}+\right.$$

$$\left.+\frac{1}{2\pi i}\int e^{st}\frac{s+\alpha-\sqrt{s^2+2\alpha s}}{\sqrt{s^2+2\alpha s}}ds\right\}=$$

$$=-\frac{B}{\varepsilon}e^{-t/\tau}\{e^{-\xi}I_0(\xi)+e^{-\xi}I_1(\xi)\}=$$

$$=-\frac{B}{2\varepsilon}\exp\left[-\frac{1}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)t\right]\{I_0(\xi)+I_1(\xi)\},$$

$$\xi=\frac{1}{2}\left(\frac{1}{\tau_1}-\frac{1}{\tau}\right)t>0 \qquad (3.26)$$

We use asymptotic formulas for $x\gg 1$:

$$I_0(x)=\frac{e^x}{\sqrt{2\pi x}}\left[1+\frac{1}{8x}+O\left(\frac{1}{x^2}\right)\right],$$

$$I_1(x)=\frac{e^x}{\sqrt{2\pi x}}\left[1-\frac{3}{8x}+O\left(\frac{1}{x^2}\right)\right] \qquad (3.27)$$

These relations give us the asymptotes to the solution to (3.24)

$$v(L_0\to\infty,\ t\to\infty)=$$

$$=-\frac{B}{2\varepsilon\sqrt{\pi}}e^{-t/\tau_1}\left[\left(\frac{1}{\tau}-\frac{1}{\tau_1}\right)t\right]^{-1/2}\quad\text{for }\tau_1>\tau,$$

$$v(L_0\to\infty,\ t\to\infty)=$$

$$=-\frac{B}{\varepsilon\sqrt{\pi}}e^{-t/\tau}\left[\left(\frac{1}{\tau_1}-\frac{1}{\tau}\right)t\right]^{-1/2}\quad\text{for }\tau_1<\tau. \qquad (3.28)$$

Let $\tau=\tau_1$ (the Heaviside case). If then $R_1=R^*$, the integral of (3.19) can be transformed to

$$v(L_0,t)=\frac{1}{2}f(t)-\frac{R^*}{R_0+R^*}e^{-t_0/\tau}f(t-t_0)\,\eta(t-t_0)+$$

$$+\frac{R^*-R_0}{2(R_0+R^*)}e^{-2t_0/\tau}f(t-2t_0)\,\eta(t-2t_0), \qquad (3.29)$$

in which $t_0=L_0/c_1$ is the time taken by a signal to travel along a cable of length L_0:

$$f(t)=\frac{1}{2\pi i}\int\limits_{a-i\infty}^{a+i\infty}e^{pt}\frac{F_1(p)}{p+1/\tau}dp=-\frac{B}{\varepsilon}e^{-t/\tau}\int\limits_0^t J(t')\,e^{t'/\tau}dt',$$

$$\eta(t<0)=0,\quad \eta(t>0)=1. \qquad (3.30)$$

Still taking $\tau=\tau_1$, we abandon $R_1=R^*$ and put

$$p+\frac{1}{\tau}=\frac{z}{t_0},\quad \xi=\frac{t}{t_0},$$

$$\lambda_{0,1}=\frac{R_{0,1}-R^*}{R_{0,1}+R^*},\quad Q=(\lambda_0\lambda_1)^{-1}, \qquad (3.31)$$

so $|\lambda_{0,1}|<1$, $|Q|>1$, and for pulse irradiation we get

$$v(L_0,t)=-\frac{BR_1e^{-t/\tau}}{\varepsilon(R_0+R^*)(R_1+R^*)}\frac{1}{2\pi i}\times$$

$$\times\int\limits_{a-i\infty}^{a+i\infty}e^{z\xi}\frac{(R_0+R^*)\,e^{2z}-2R^*e^z+R^*-R_0}{e^{2z}-Q^{-1}}\frac{dz}{z}. \qquad (3.32)$$

The point $z=0$ is not singular. For $Q>0$ the first-order poles lie at

$$z_n=-{}^1\!/_2\ln Q+i\pi n\quad(n=0,\pm1,\pm2,\ldots). \qquad (3.33)$$

We replace the integral of (3.32) by the sum of the residues:

$$v(L_0,t)=-\frac{BR_1e^{-t/\tau}}{\varepsilon(R_1+R^*)}Q^{-\xi/2}\times$$

$$\times\operatorname{Re}\left\{(\sqrt{Q}+\lambda_0)(\sqrt{Q}-1)\sum_{n=0}^{\infty}\frac{e^{i2\pi n\xi}}{{}^1\!/_2\ln Q-i2\pi n}+\right.$$

$$\left.+(\lambda_0\sqrt{Q}-1)(\sqrt{Q}+1)\sum_{n=0}^{\infty}\frac{e^{i\pi(2n+1)\xi}}{{}^1\!/_2\ln Q-i\pi(2n+1)}\right\}. \qquad (3.34)$$

If $Q<0$, the poles lie at

$$z_n=-{}^1\!/_2\ln|Q|+i\pi(n+{}^1\!/_2)\quad(n=0,\pm1,\pm2,\ldots). \qquad (3.35)$$

The integral of (3.32) then becomes the sum

$$v(L_0,t)=\frac{BR_1e^{-t/\tau}}{\varepsilon(R_1+R^*)}|Q|^{-1/2\,\xi}\times$$

$$\times\operatorname{Re}\left\{\sum_{n=0}^{\infty}\frac{e^{i\pi(n+1/2)\xi}}{{}^1\!/_2\ln|Q|-i\pi(n+{}^1\!/_2)}\times\right.$$

$$\left.\times\left[1+\frac{2iR^*}{R_0+R^*}|Q|^{1/2}+\lambda_0|Q|\right]\right\}. \qquad (3.36)$$

It is convenient to put (3.19) with $J(t)=\delta(t)$ as follows for some purposes:

$$v(L_0t)=-\frac{BR_1}{2\pi i\varepsilon}\exp\left[-\frac{1}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)t\right]\int\limits_{a-i\infty}^{a+i\infty}\frac{e^{\xi z}dz}{z+\omega}\times$$

$$\times\frac{\operatorname{ch}\theta-1+(R_0/R^*)\,\Omega\operatorname{sh}\theta}{(R_0+R_1)\operatorname{ch}\theta+[(R_0R_1/R^*)\,\Omega+R^*\Omega^{-1}]\operatorname{sh}\theta} \qquad (3.37)$$

in which

$$p+\frac{1}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)=\frac{1}{2}\left|\frac{1}{\tau}-\frac{1}{\tau_1}\right|z,\quad \xi=\frac{t}{2}\left|\frac{1}{\tau}-\frac{1}{\tau_1}\right|,$$

$$\nu=\frac{t_0}{2}\left|\frac{1}{\tau}-\frac{1}{\tau_1}\right|,\quad t_0=\frac{L_0}{c_1}\qquad \theta=\alpha L_0=\nu\sqrt{z^2-1},$$

$$\omega=\left(\frac{1}{\tau}-\frac{1}{\tau_1}\right)\Big/\left|\frac{1}{\tau}-\frac{1}{\tau_1}\right|,\quad \Omega=\left(\frac{z+\omega}{z-\omega}\right)^{1/2}. \qquad (3.38)$$

We perform the transformation

$$z=\frac{1}{2}\left(w-\frac{1}{w}\right),\quad w=z+\sqrt{z^2-1},$$

$$\sqrt{z^2-1}=\frac{1}{2}\left(w-\frac{1}{w}\right),\quad z\pm1=\frac{(w\pm1)^2}{2w}, \qquad (3.39)$$

which transfers the outer part of $-1<z<1$ to the exterior of unit circle $|w|>1$. Let initially $\tau_1>\tau$, i.e., $\omega=+1$. Then

$$v(L_0, t) = \frac{-BR_1}{2\pi i\varepsilon(R_1 + R^*)}\exp\left[-\frac{t}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)\right]\times$$

$$\times\int_{a-i\infty}^{a+i\infty}\exp\left[\frac{\xi}{2}\left(w+\frac{1}{w}\right)\right]\frac{w-1}{w}\times$$

$$\times\frac{(e^\theta - 1)[(w+\lambda_0)e^\theta + \lambda_0 w + 1]\,dw}{(w+\lambda_0)(w+\lambda_1)e^{2\theta} - (w\lambda_0+1)(w\lambda_1+1)}. \quad (3.40)$$

Here λ_0 and λ_1 are as in (3.31). The integral of (3.40) can also be calculated along a vertical line to the right of the poles in the integrand. The integrand in (3.37) is of one sheet, so the integral of (3.40) equals zero as taken on the circle $|w| = 1$ (which corresponds to double passage on $-1 < z < 1$ in the z plane), and so in (3.40) we need take account only of poles lying outside the unit circle $|w| = 1$. To find the poles $w = \rho e^{i\varphi}$ we have

$$\exp\left\{\nu\left[\left(\rho-\frac{1}{\rho}\right)\cos\varphi + i\left(\rho+\frac{1}{\rho}\right)\sin\varphi\right]\right\} =$$

$$= \frac{\lambda_0\rho e^{i\varphi}+1}{\rho e^{i\varphi}+\lambda_0}\,\frac{\lambda_1\rho e^{i\varphi}+1}{\rho e^{i\varphi}+\lambda_1}. \quad (3.41)$$

If $\rho > 1$ ($\rho < 1$), the right part of this equation is less (greater) in modulus than unity, which means that $\cos\varphi < 0$, i.e., all roots of (3.41) lie in the left half-plane. The semicircle $w = e^{i\varphi}$, $-\pi/2 < \varphi < \pi/2$ corresponds to double passage in $0 < z < 1$, so the integral of (3.40) along this semicircle is zero. Then the integral of (3.40) can be calculated along Re $w = 0$ except the part joining the points $-i$ and i:

$$v(L_0, t) = -\frac{BR_1}{\pi\varepsilon(R_1+R^*)}\exp\left[-\frac{t}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)\right]\times$$

$$\times\,\mathrm{Re}\int_1^\infty e^{\frac{i\xi}{2}\left(x-\frac{1}{x}\right)}\left(x+\frac{i}{x}\right)\times$$

$$\times\frac{(e^\theta-1)[(ix+\lambda_0)e^\theta + i\lambda_0 x + 1]\,dx}{(ix+\lambda_0)(ix+\lambda_1)e^{2\theta} - (i\lambda_0 x+1)(i\lambda_1 x+1)}$$

$$\theta = \frac{i\nu}{2}\left(x+\frac{1}{x}\right). \quad (3.42)$$

Now let $\tau_1 < \tau$, i.e., $\omega = -1$; formula (3.37) transforms to

$$v(L_0, t) = -\frac{BR_1}{\varepsilon(R_1+R^*)}\exp\left[-\frac{t}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)\right]\times$$

$$\times\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\exp\left[\frac{\xi}{2}\left(w+\frac{1}{w}\right)\right]\frac{w+1}{w}\times$$

$$\times\frac{(e^\theta-1)[(w-\lambda_0)e^\theta+\lambda_0 w - 1]\,dw}{(w-\lambda_0)(w-\lambda_1)e^{2\theta} - (w\lambda_0-1)(w\lambda_1-1)}. \quad (3.43)$$

The denominators in (3.40) and (3.43) differ only in the signs to λ_0 and λ_1, and the integral of (3.43) may be put as

$$v(L_0, t) = -\frac{BR_1}{\pi\varepsilon(R_1+R^*)}\exp\left[-\frac{t}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)\right]$$

$$\mathrm{Re}\int_1^\infty \exp\left[\frac{i\xi}{2}\left(x-\frac{1}{x}\right)\right]\left(1-\frac{i}{x}\right)\times$$

$$\times\frac{(e^\theta-1)[(ix-\lambda_0)e^\theta + i\lambda_0 x - 1]\,dx}{(ix-\lambda_0)(ix-\lambda_1)e^{2\theta} - (i\lambda_0 x - 1)(i\lambda_1 x - 1)},$$

$$\theta = \frac{i\nu}{2}\left(x+\frac{1}{x}\right). \quad (3.44)$$

Returning to (3.37), we see that we may take the imaginary axis Re $z = 0$ as the line of integration, so the integral of (3.37) may be put as

$$v(L_0, t) = -\frac{B}{\pi\varepsilon R_0}\exp\left[-\frac{t}{2}\left(\frac{1}{\tau}+\frac{1}{\tau_1}\right)\right]\mathrm{Re}\int_0^\infty e^{i\xi x}\times$$

$$\times\frac{R_0(ix+\omega)\sin\nu y - R^*(1-\cos\nu y)\,y}{(R^*/R)\,y\cos\nu y + [(1+\gamma)ix + (1-\gamma)\omega]\sin\nu y}\,\frac{dx}{ix+\omega},$$

$$y = \sqrt{x^2+1},\quad \gamma = (R^*)^2(R_0R_1)^{-1},\quad \frac{1}{R} = \frac{1}{R_0}+\frac{1}{R_1}. \quad (3.45)$$

From (3.19) we readily find that, if $R_1 = R^*$,

$$v(L_0, t)|_{R_0=\infty} = v(2L_0, t)|_{R_0=R^*}. \quad (3.46)$$

We note by $\psi(t)$ the solution $v(L_0, t)$ for $R_0 = 0$, $R_1 = R^*$. We now assume (purely formally) that we have a cable of length $2L_0$, with the parts $0 < x < L_0$ and $-L_0 < x < 0$ having equal polarizations opposite in sign. It can be shown that matched loads $R_0 = R_1 = R^*$ at $x = \pm L_0$ cause the voltage at $x = L_0$ to be $\psi(t)$. The physical significance of these relationships is obvious.

Finally we consider a thin cable, for which $\delta \gg r_2$, and we can assume that the fast electrons formed at any point in the dielectric move in the direction of the γ-ray and strike the opposite metal surface. We also assume that the range of the Compton electrons in the metal is so small that we can neglect escape from the core and sheath. Here the volume density of the absorbed γ-rays can be found by replacing exp in (1.3) and (1.6) by unity:

$$n_\gamma = \frac{J}{\lambda_0}, \quad (3.47)$$

where J denotes the total current, as in section 1. Since all the fast electrons strike either the core or the sheath, the dielectric has a positive charge, and (1.8) gives the charge density as

$$\rho_0 = \frac{e_0 J}{N\lambda_0}. \quad (3.48)$$

Unit length of the internal cylinder has the negative charge

$$q_0 = -\rho_0 S = -\frac{e_0 S}{N\lambda_0}J, \quad (3.49)$$

in which S is the area of the region in Fig. 1 defined by $x < 0$, $|y| < r_1$, $r_1 < r = (x^2+y^2)^{1/2} < r_2$. With $\sin\alpha = r_1/r_2$ we have

$$S = r_2^2[\alpha + \sin\alpha\cos\alpha - {}^1\!/_2\,\pi(\sin\alpha)^2]. \quad (3.50)$$

The positive space charge of (3.48) and the negative charge of (3.49) give a field having the potential

$$\psi(r) = -\frac{\pi}{\varepsilon}\rho_0 r^2 + C_1\ln r + C_2. \quad (3.51)$$

in which

$$C_1 = \frac{2e_0 r_2^2}{\varepsilon N\lambda_0}\left[\alpha + \sin\alpha\cos\alpha + \frac{\pi}{2}(\sin\alpha)^2\right]J. \quad (3.52)$$

The potential difference $V = \psi(r_2) - \psi(r_1)$ between the metal parts may, by analogy with (1.25), be written as

$$V = \frac{B^*}{\varepsilon}J,$$

$$B^* = -\frac{e_0\pi r_2^2}{N\lambda_0}\left[\cos^2\alpha - \left(\frac{2\alpha+\sin 2\alpha}{\pi}+\sin^2\alpha\right)\ln\left(\frac{1}{\sin\alpha}\right)\right]. \quad (3.53)$$

All the remaining theory for thin cables may be derived from the above if the B of (1.25) is everywhere replaced by the B^* of (3.53).

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